Q.P. code :- 58653

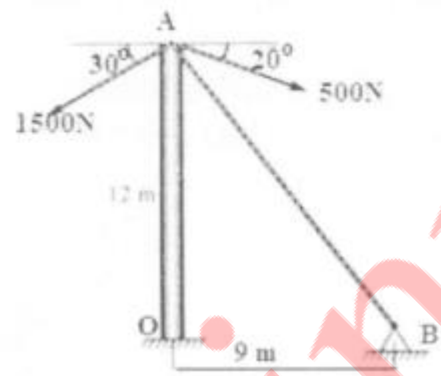
ENGINEERING MECHANICS – SEMESTER – 1

CBCGS DEC 2019

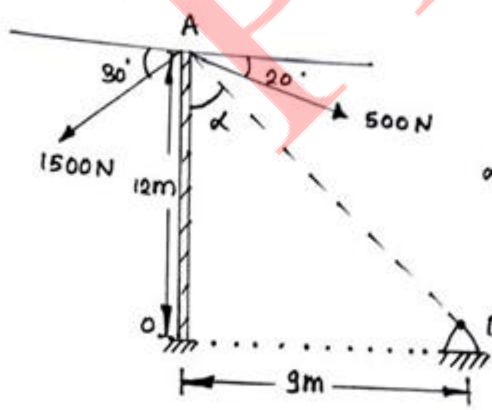
1.)

a. **(4)**

The top end of a pole is connected by three cables having. tension 500 N ,1500 N and a guy wire 'AB' as shown in figure below. Determine tension in cable 'AB' if the resultant of the concurrent forces is vertical.



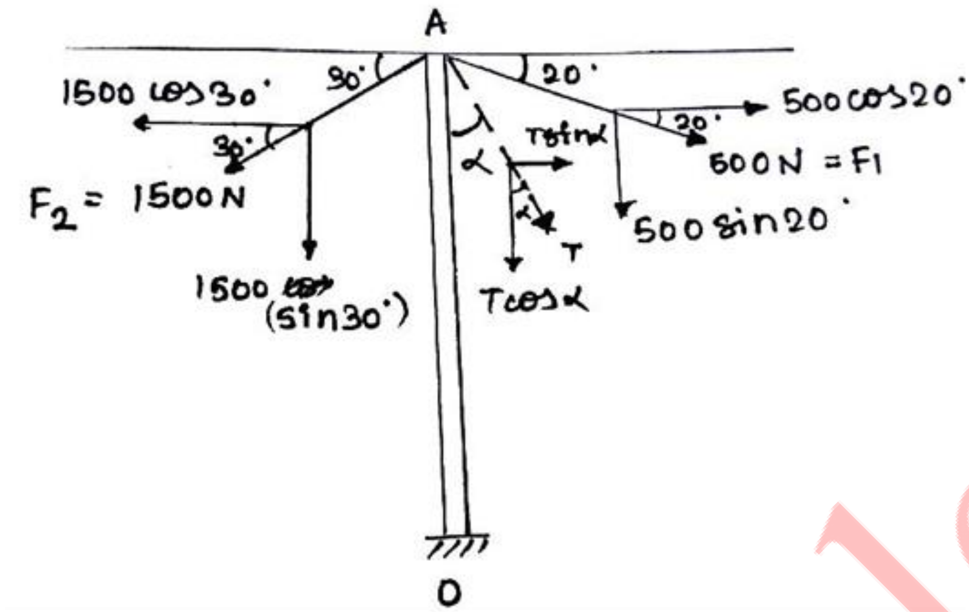
Soln:-



In Δ AOB

$\alpha = \tan^{-1}(9/12)$

$= 36.86°$



Given:- In the figure , F1=500N,F2=1500N,T=?,F1 makes an angle 20° with horizontal and F2 makes 30° with the horizontal and assume tension ‘T’ makes an angle α with vertical.

Resultant force in vertical direction.

To find:- Tension ‘T’=?

CALCULATION :-

IN ΔAOB

$\alpha = 36.86°$

Taking forces having direction towards right as positive and forces having direction upwards as

Positive.

Resolving forces along X direction :

$Rx = F\ 1\ \cos20° – F\ 2\ \cos30° + T\sin\alpha$

$= 500\cos20° – 1500\cos30° + T\sin\alpha$ ....(1)

Resolving forces along Y direction:

$Ry = -F\ 1\ \sin20° - F\ 2\ \sin30° - T\ \cos\alpha$

$= -500\sin 20° - 1500\sin 30° - T\cos\alpha$

..........(2)

Resultant force is in upward direction so

Rx=0

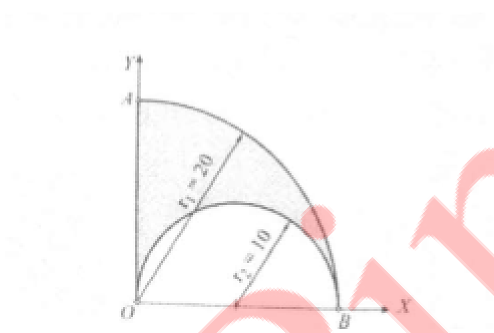
Put Rx = 0 ,α=36.86°(calculated) in equation 1

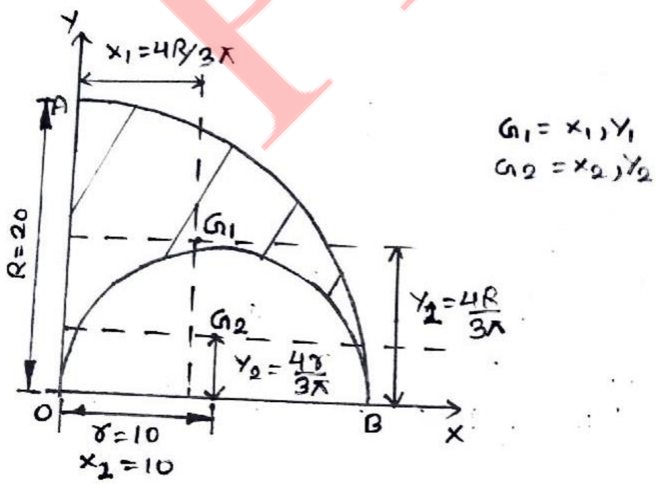
$500\cos 20° - 1500\cos 30° + T\sin 36.86° = 0$

T = 1382.304N (ANS)

b. **(4)**

Locate the centroid of the shaded area obtained by cutting a semicircle of diameter 20mm from quadrant of a circle of radius 20mm as shown in figure below.





Soln:-

Given:- Co-ordinates of shaded portion can be obtained by taking a quater circle of radius 20mm and subtracting a semi-circle of radius 10mm.

To find:- centroid co-ordinates.

Calculation:-

PART	Area (Ai) mm^2	Xi mm	Yi mm	Aixi mm^3	Aiyi mm^3
1.Quater circle	314.15	8.488	8.488	2666.50	2666.50
2.semi-circle	-157.08	10	4.244	-1570.8	-666.64

X co-ordinate of centroid ($\bar{x}$) = ΣAxi/ΣA = 1095.7/157.07 = 6.97mm

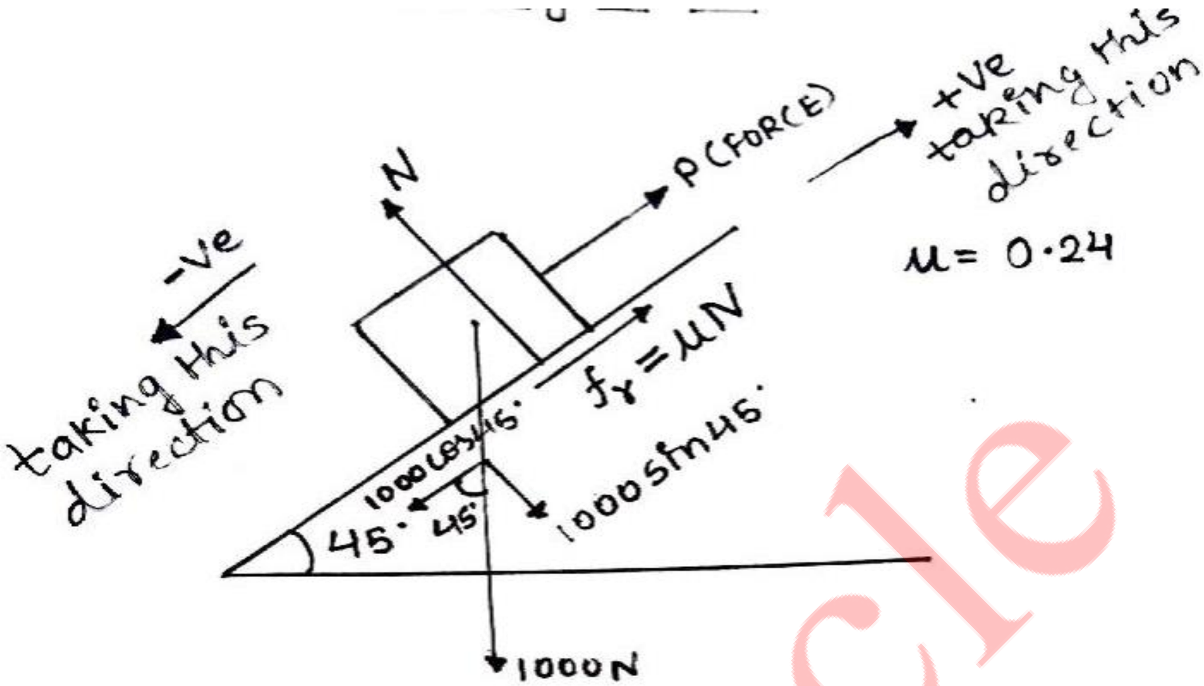
Y co-ordinate of centroid($\bar{y}$) =ΣAyi/ΣA = 1999.86/157.07 = 12.73mm

Centroid is at (6.97,12.73)mm (ANS)

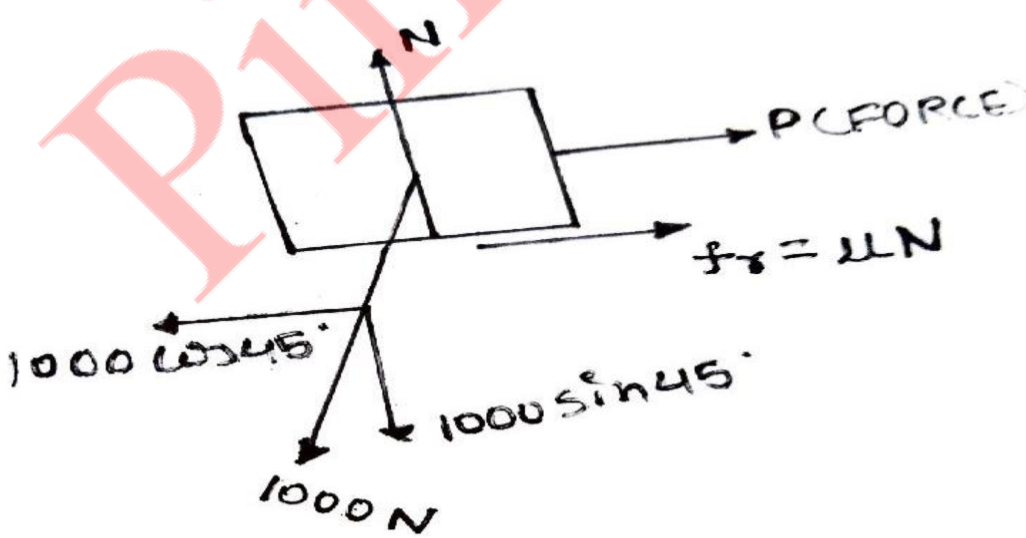
c. **(4)**

A body weighing 1000N is lying on a horizontal plane. Determine necessary force to move the body along the plane if the force is applied at an angle of 45 degrees to the horizontal with coefficient of friction 0.24.

Soln:-



FBD OF BLOCK :-



Let the normal force be 'N' and friction force be 'fr' and force 'P' be the force required to keep the body in equilibrium and +ve as X-axis and –ve as Y-axis .

Applying equilibrium conditions on Block,

$\Sigma F_X = 0$

$P + fr - 1000\cos(45°) = 0$ ....(1)

$\Sigma F_Y = 0$

$N - 1000\sin(45°) = 0$ ....(2)

N = 707.106 N (put this in equation (1))

$P + \mu N - 1000\cos(45°) = 0$ ($fr = \mu N, \mu = 0.24$)

$P = 1000\cos(45°) - 0.24*(707.106)$

The minimum weight of P is

P = 537.40N (ANS)

d. **(4)**

The motion of the particle is defined by the relation x = t^3-3t^2+2t+5 where x is the position expressed in meters and time in seconds. Determine (i) the velocity and acceleration after 5 seconds. (ii)maximum or minimum velocity and corresponding displacement.

Soln:-

Given :- Rightward as +ve and Leftward as –ve.

x(t)= t^3-3t^2+2t+5

$v(t) = dx/dt$

$= 3t^2-6t+2$

$a(t) = dv/dt$

$= 6t-6$

(i) $v(5) = 3(5)^2-6(5)+2$

$= 47 m/s^2$

$a(5) = 6(5)-6$

$= 24 m/s^2$

(ii) maximum or minimum velocity and corresponding displacement,happens only when $dv/dt = 0$. Put $dv/dt = 0$

$6t-6=0$

$t=1$

If $d2v/dt2$ is positive then their will be minima otherwise maxima.

$d2v/dt2 = 6$ (positive)

So, minimum velocity exist .

At $t=0$,

$v(1) = 3(1)^2-6(1)+2$

$= -1$ or $1 m/s^2$ (in left direction)

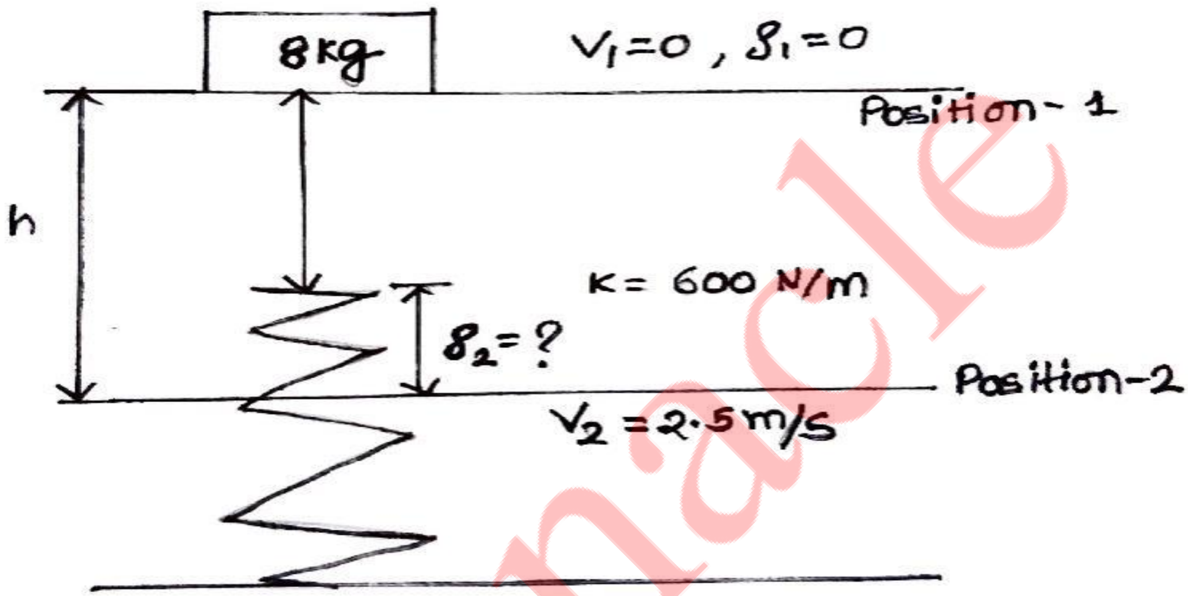
$x(1)=(1)^3-3(1)^2+2(1)+5$

$= 5m$ **(ANS)**

e. **(4)**

A steel ball of mass 8 kg is dropped onto a spring of stiffness 600 N/m and attains a maximum velocity 2.5 m/s. Find (i) the height from it is dropped and (ii) the maximum deflection of spring.

Soln:-



Given:- The fall is free and the starts with zero velocity ,

At position 1 ,the velocity is zero , delection is zero ,K= 600N/m , mass of body = 8Kg ,

At position 2 , the velocity is 2.5m/s^2 and deflection say 'δ',

Work done :-

(i work done by weight = mgh

= 8*9.81*h (v^2=u^2+2g(h- δ2)

(this equation is applied for free fall body so height will be h- δ2 because after that motion is influenced by spring.)

$((2.5)^2=0+2*9.81*(h-\delta_2))$

$(h=(0.31855+\delta_2)\ m)$

$= 8*9.81*(0.31855+\delta_2)$

$=(25+78.48\delta_2)\ J$

(ii work done by spring = $\frac{1}{2}*k*((\delta_1)^2 - (\delta_2)^2)$

$= \frac{1}{2}*(600)*(0-(\delta_2)^2)$

$= -300(\delta_2)^2\ J$

summation of all work done = ΣU_{1-2}

$= (25+78.48\delta_2) - 300(\delta_2)^2$

BY WORK ENERGY THEOREM:-

$T_1 + \Sigma U_{1-2} = T_2$

T1= INITIAL KINETIC ENERGY = $\frac{1}{2}*m*v_1^2 = \frac{1}{2}*8*0$

T2 = FINAL KINETIC ENERGY = $\frac{1}{2}*m*v_2^2 = \frac{1}{2}*8*(2.5)^2 = 25$

$0 + (25+78.48\delta_2) - 300(\delta_2)^2 = 25$

$(\delta_2) = 0.2616$ or 0

$(\delta_2) = 0.2616$ m (maximum deflection)(ii ans)

$h= (\delta_2) + 0.31855$

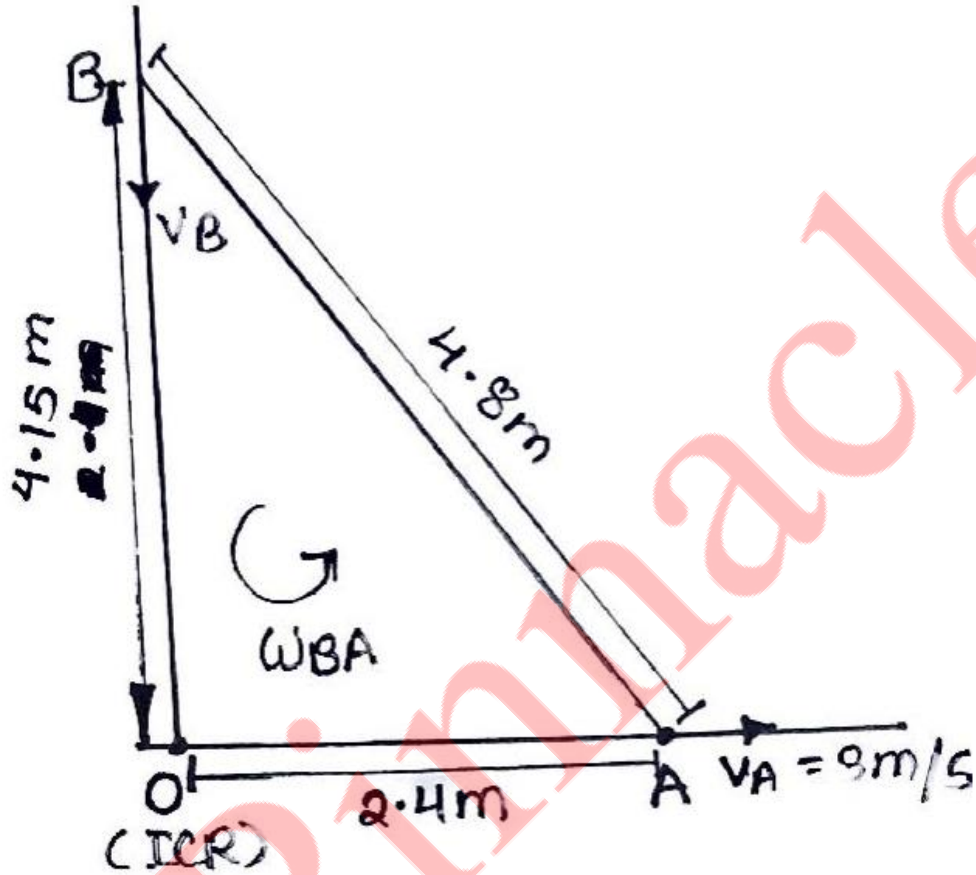
= 0.58015 m (height from where it is dropped)(i ans)

f. **(4)**

A ladder AB of length l=4.8 m rests on a horizontal floor at A and leans against a vertical wall at B. If the lower end A is pulled away

from the wall with a constant velocity 3 m/s, what is the angular velocity of the ladder at the instant when A is 2.4 m from wall.

Soln:-



Given :- In **ΔAOB ,** OA and AB is given in the problem. Velocity of A is 3m/s and at b is unknown. Point of rotation or instantaneous center of rotation is O. Rotation is from B to A.

To find:- ω_{BA} = ?

Calculation:- In **ΔAOB,** OA and AB are 2.4m and 4.8m resp., By pythagoras theorem ,

OB = √AB^2 - OA^2

$= \sqrt{4.8^2 - 2.4^2}$

$= 4.15m$

Instantaneous center of rotation is the point of intersection of

vA and vB velocity vector.

Radius of rotation is perpendicular distance of velocity vector of point from instantaneous center of rotation (ICR).

So, $r_A = 2.4m$

$r_B = 4.15m$

Instantaneous velocity of $v_A = \omega_{BA} * r_A$.

$3 = \omega_{BA} * (2.4)$

Angular Velocity of motion from B to A = ω_{BA} = 1.25 rad/sec (ANS)

Instantaneous velocity of $v_B = \omega_{BA} * r_B$

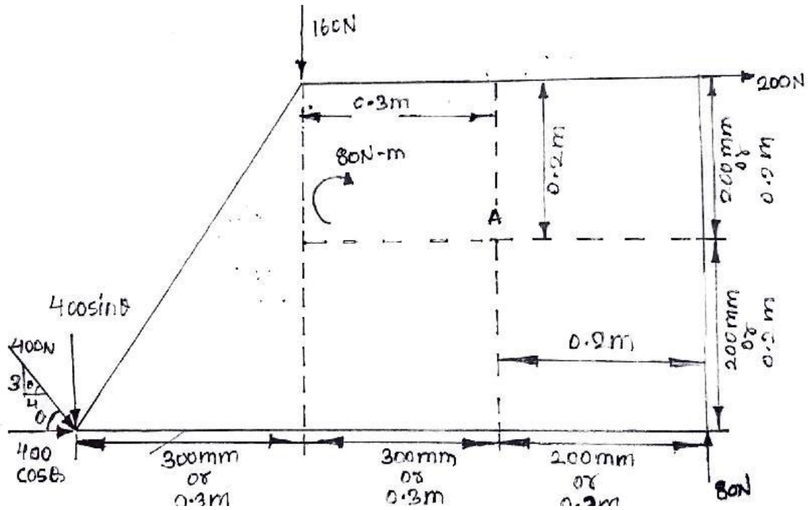
$v_B = 1.25 * 4.15$

$= 5.18 m/s$

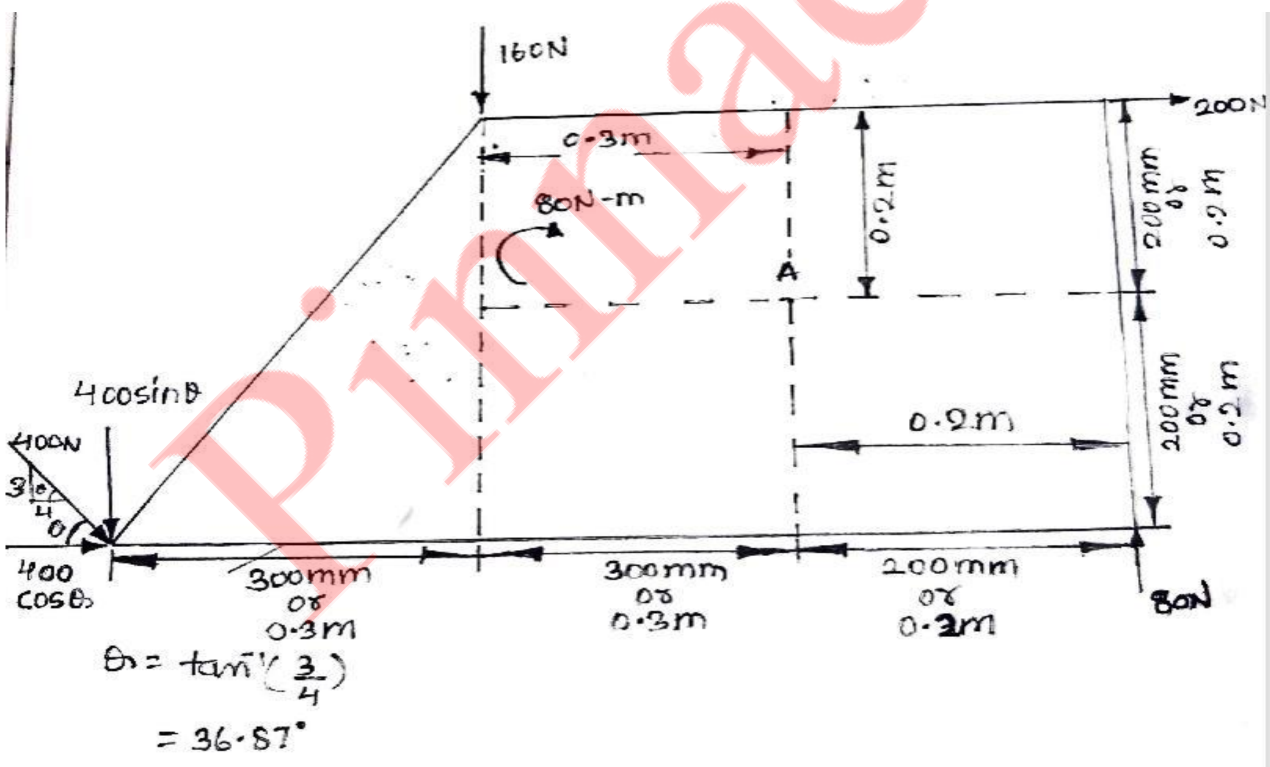
2.)

a. **(8)**

Find the resultant of the force system acting on the plate as shown in figure, where does this resultant act with respect to point A ?



Soln:-



Given:- The figure 1st and 2nd .

To find:- R**esultant act with respect to point A**

Calculation:-

$\Sigma Fx = 200 + 400\cos(36.87°)$ (taking right as +ve)

$= +519.99$ N or 519.99 N (rightwards)

$\Sigma Fy = 80 – 160 – 400\sin(36.87°)$ (taking upwards as +ve)

$= -320.01$ N or 320.01 N (downwards)

R = √(Fx)^2 + (Fy)^2

= √(519.99)^2 + (320.01)^2

= 610.57 N

ϴ = tan-1(Fy/Fx)

= tan-1(320.01/519.99)

= 31.60°

MOMENT OF ALL THE FORCES ABOUT POINT 'A'.

MOMENT = F*(Perpendicular distance of line of force from the point)

$\Sigma M_A = -200*(0.2) + 80*(0.2) + 160*(0.3) - 400\sin(36.87°)*(0.6)$

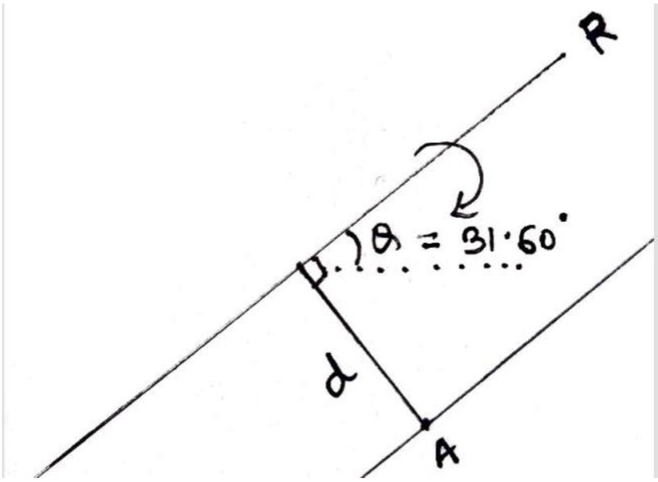
(taking anti-clockwise as +ve)

= -120N-m or 120N-m(clockwise)

$\Sigma M_A = R*d$

120 = 610.57*d

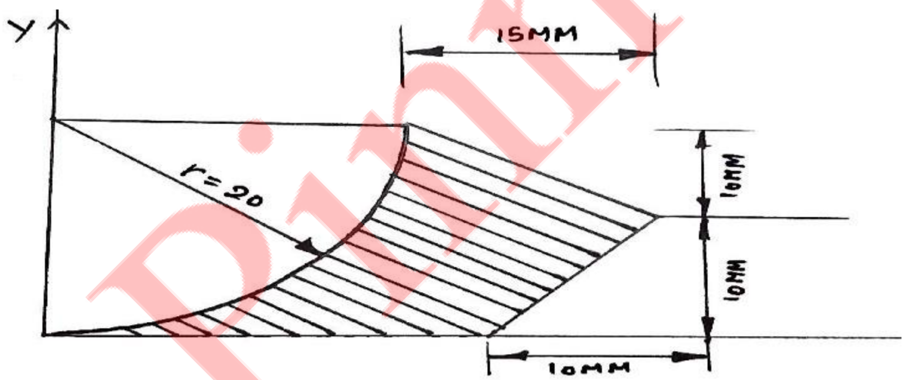
d= 0.1965 m



THE RESULTANT OF FORCE IDS ACTING CLOCKWISE ABOUT POINT 'A' AND AT A DISTANCE OF d = 0.196m or 196mm. (ans)

b. **(6)**

Find centroid of the shaded area with reference to X and Y Axes.



Soln:-

Given:- G1,G2,G3,G4 are centroids of respective figures.

Area of the shaded region = Rectangle ACHE - Quarter Circle ABC – Triangle BHF - Triangle DEF

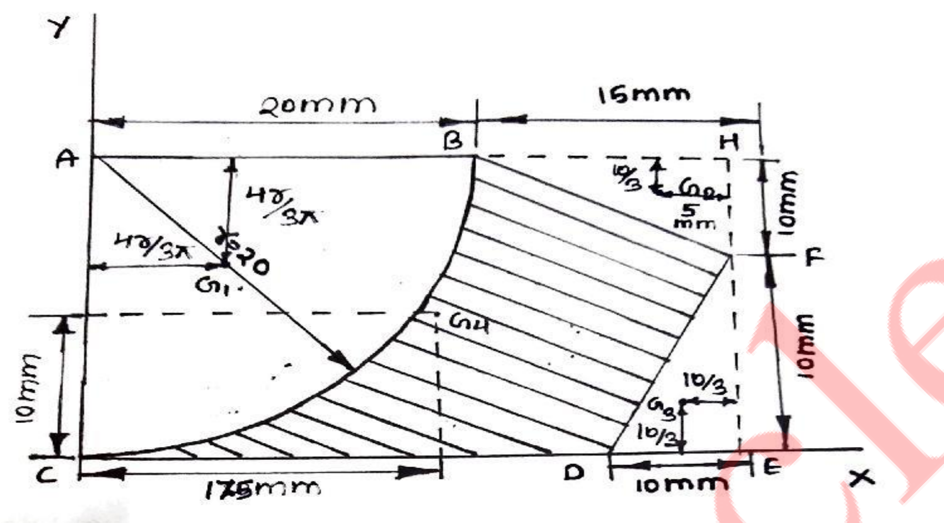


Figure	AREA (mm^2)	X-coordinate (mm)	Y-coordinate (mm)	Aixi (mm^3)	Aiyi (mm^3)
Rectangle ACHE	35*20 = 700	17.5	10	12250	7000
Quarter circle ABC	-¼*Π*r^2 = -314.15	4r/3Π = 4*20/3Π =8.48	20- 4r/3Π = 20- 4*20/3Π = 11.52	-2663.99	-3619.008
Triangle BHF	-½*10*15 =-75	35-5=30	20-10/3 =16.67	-2250	-1250.25
Triangle DEF	-½*10*10 =-50	35-10/3 = 31.67	10/3 = 3.33	-1583.5	-166.5

∑Ai = 700 – 314.15 - 75 – 50 = 260.85 mm^2

∑Aixi = 12250 – 2663.99 - 2250 – 1583.5 = 5752.51 mm^3

∑Aiyi = 7000 – 3619.008 - 1250.25 - 166.5 = 1964.242 mm^3

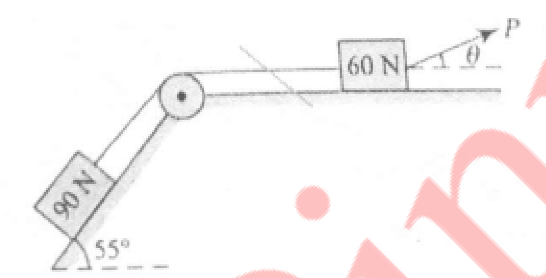
$\bar{x}$ = ∑Aixi/∑Ai = 5752.51/260.85 = 22.05mm

$\bar{y}$ = ∑Aiyi/∑Ai = 1964.242/260.85 = 7.53mm

Centroid coordinates of lamina is (22.05mm, 7.53mm) (ANS)

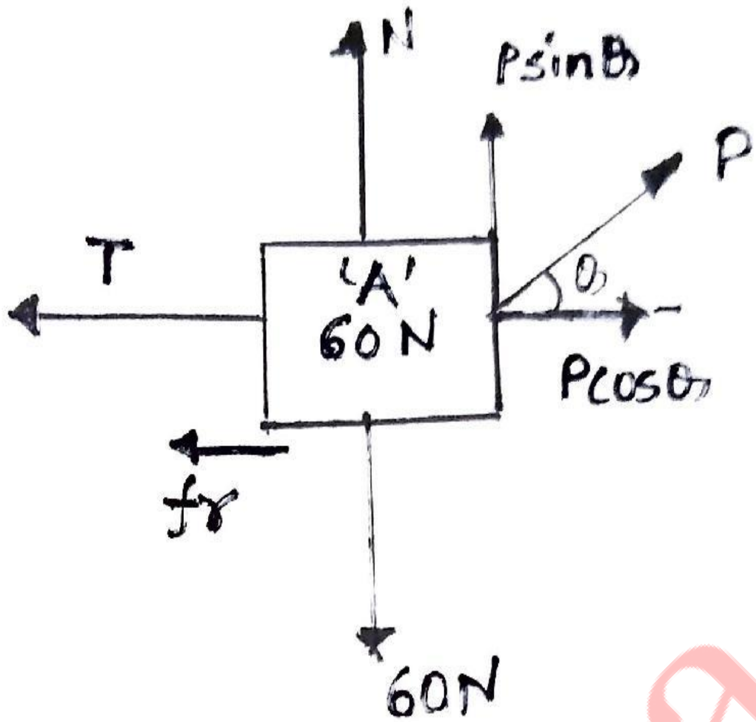
c. **(6)**

Two bodies A and B weighing 90 N and 60 N respectively placed on an inclined plane are connected by the string which is parallel to the plane as shown in Fig. Find the inclination of the minimum force P for the motion to impending the direction of "p". Take μ = 0.2 for the surface of contact.



Soln:-

FBD of 60N block:-



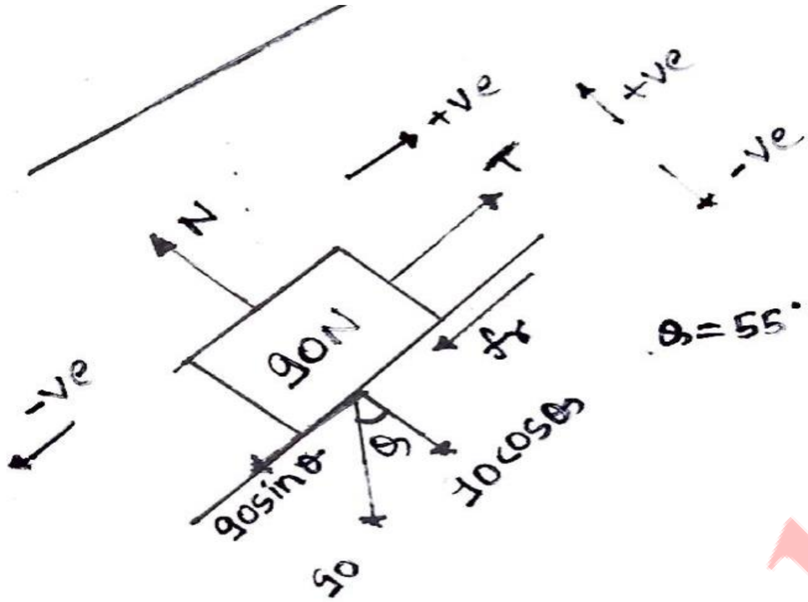
Consider block A

$\sum Fx=0$ (consider right as +ve)

$P\cos\theta - T - fr = 0$ ....(1)

$\sum Fy=0$ (Consider upward as +ve)

$P\sin\theta + N_A - 60 = 0$ ......(2)



Consider block 90N

$\sum F_x=0$ (consider right as +ve)

$T - 90\sin(55°) - fr = 0$ ....(3) ($fr = \mu * N_B$, $\mu = 0.2$)

$\sum F_y=0$ (Consider upward as +ve)

$N_B - 90\cos(55°) = 0$ ......(4)

$N_B = 51.62$ N (put in equation 3)

$T = 90\sin(55°) + \mu * N_B$

$= 84.04$ N (put in equation 1)

$P\cos\theta - T - fr = 0$ ($fr = \mu * N_A$, $\mu = 0.2$)

$P\cos\theta - 84.04 - 0.2 * N_A = 0$

$P\sin\theta + N_A - 60 = 0$ (equation 2)

$N_A = 60 - P\sin\theta$

Put in equation 1

$P\cos\theta - 84.04 - 0.2*(60 - P\sin\theta) = 0$

$P\cos\theta - 84.04 - 12 + 0.2* P\sin\theta = 0$

$P\cos\theta + 0.2* P\sin\theta = 96.04$

$P = 96.04/(\cos\theta + 0.2*\sin\theta)$

To minimize P, differentiate then equate to zero

$dP/d\theta = -96.04*(-\sin\theta + 0.20\cos\theta)/(\cos\theta + 0.20\sin\theta)^2 = 0$

$-\sin\theta + 0.20\cos\theta = 0$

$\sin\theta = 0.20\cos\theta$

$\tan\theta = 0.20$

$\theta = 11.31^\circ$

Thus,

$P_{min} = 96.04/(\cos 11.31^\circ + 0.20\sin 11.31^\circ)$

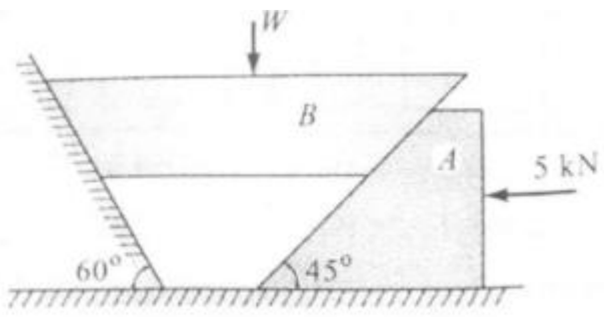
$P_{min} = 94.174$ kN

(*answer*)

3.)

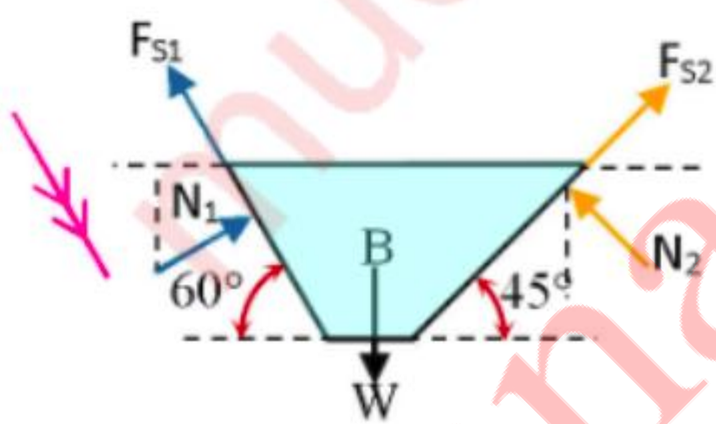
a. **(8)**

A horizontal force of 5KN is acting on the wedgeas shown in figure. The coefficient of friction at all rubbing surfaces is 0.25. Find the load "W" which can be held in position. The weight of block "B" may be neglected.



Soln:-

Let N 1 , N 2 , N 3 ,be the normal reaction at the surface of contact



∴ F S1 = μ 1 N 1 = 0.25N 1 , F S2 = μ 2 N 2 = 0.25N 2 , F S3 = μ 3 N 3 = 0.25N 3 ....(1)

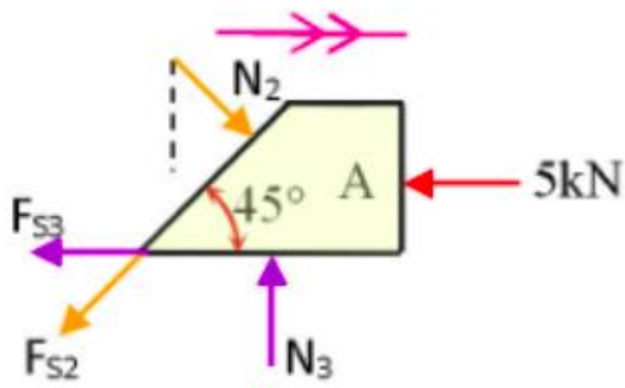
Block A is impending to move towards right.

Since the block A is under equilibrium , ∑F Y = 0

∴ N 3 − F S2 sin45 − N 2 cos45 = 0

∴ N 3 − 0.25N 2 × 0.7071 − N 2 × 7071 = 0

∴ N 3 − 0.8839N 2 = 0 ....(2)



Also $\sum F_X = 0$

$-5 - F_{S3} - F_{S2}\cos 45 + N_2 \sin 45 = 0$

................(from 1)∴ $-5 - 0.25N_3 - 0.25N_2 \times 0.7071 + N_2 \times 0.7071 = 0$ ....(from 1)

∴ $-0.25N_3 + 0.5303N_2 = 5$ ....(3)

Solving (2) and (3) simultaneously, we get $N_3 = 14.2876$kN and $N_2 = 16.1642$kN ....(4)

Block B is impending to move down

Since the block B is under equilibrium , $\sum F_X = 0$

∴ $N_1 \sin 60 - F_{S1}\cos 60 + F_{S2}\cos 45 - N_2 \sin 45 = 0$

∴ $0.866N_1 - 0.25N_1 \times 0.5 + 0.25N_2 \times 0.7071 - N_2 \times 0.7071 = 0$ ....(from 1)

∴ $0.866N_1 - 0.125N_1 + 0.1768 \times 16.1642 - 16.1642 \times 0.7071 = 0$ ....(from 4)

∴ $0.741N_1 - 8.5719 = 0$

$N_1 = 11.4939$ kN ....(5)

Also $\sum F_Y = 0$

$\therefore -W + N_1 \cos 60 + F_{S1} \sin 60 + F_{S2} \sin 45 + N_2 \cos 45 = 0$

$\therefore N_1 \times 0.5 + 0.25 N_1 \times 0.866 + 0.25 N_2 \times 0.7071 + N_2 \times 0.7071 = W$

................(from 1)

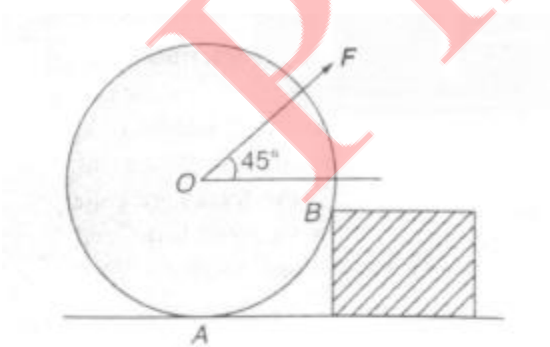
$\therefore 11.4939 \times 0.5 + 0.2165 \times 11.4939 + 0.1768 \times 16.1642 + 16.1642 \times 0.7071 = W$.....(from 4 and 5)

$\therefore W = 22.5225$kN

Hence a load of 22.5225kN can be held in the position. (ANS)

b. **(6)**

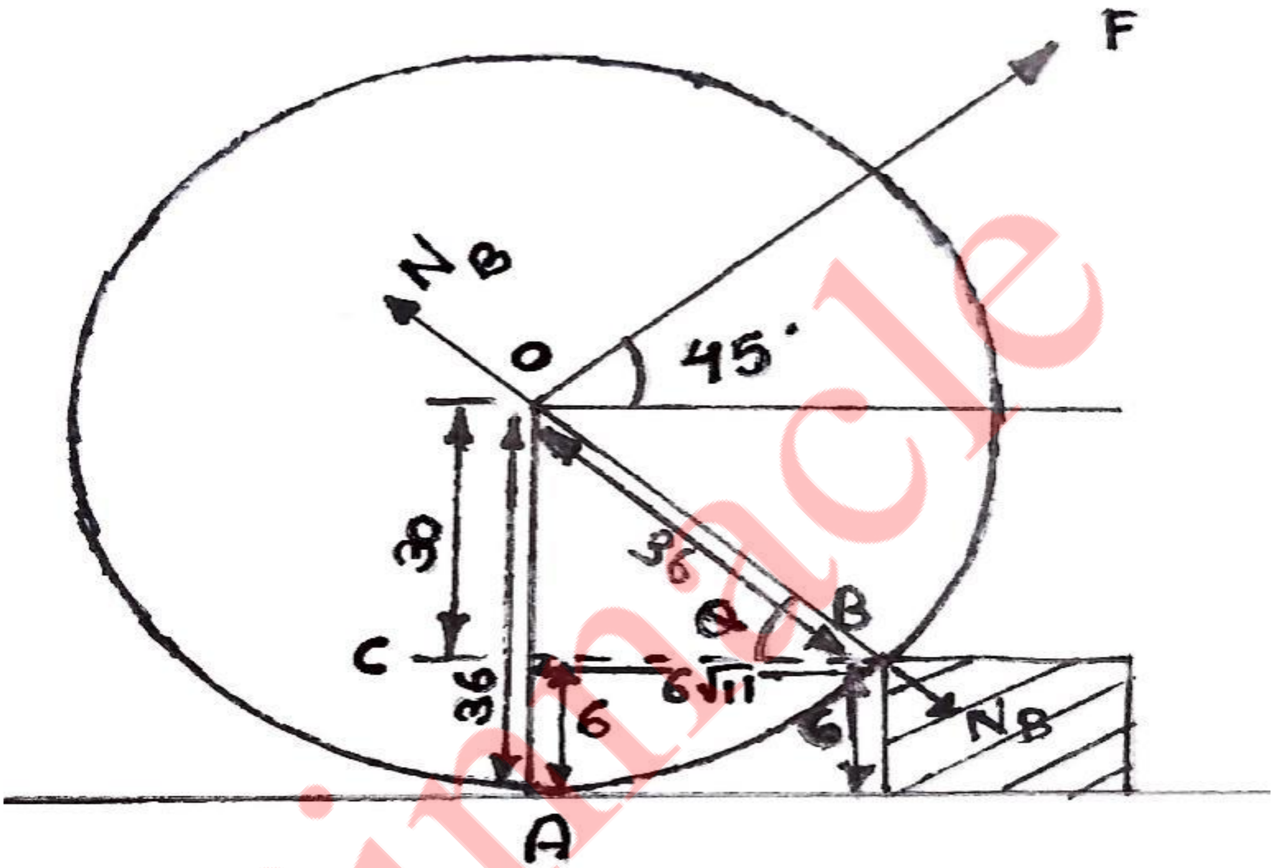
A road roller of radius 36cm and weighted 6000N, which is of cylindrical shape, is pulled by a force F, acting at an angle of 45° as shown in the figure below. It has to cross an obstacle of height 6cm. Calculate the force "F" required to just cross over the obstacle.



Soln:-

TO find the angle 'ϴ' some construction are done in the figure.

In ΔCOB, CB = √(36)^2 - (30)^2 = 6√11 cm



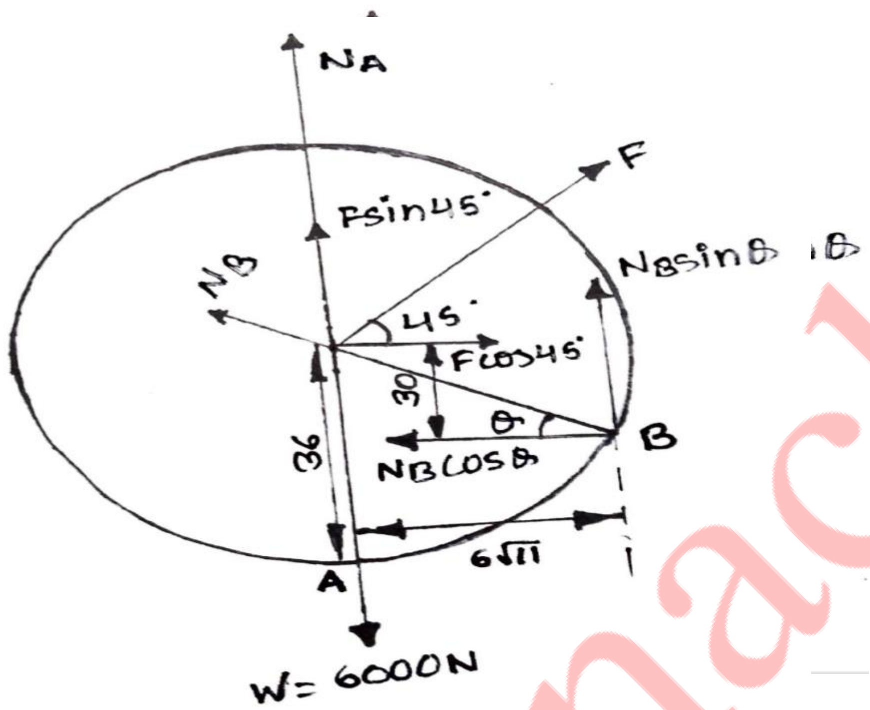
ϴ = sin-1(30/36) = 56.44°

N_B is the normal reaction between block and cylinder passing through center of cylinder.

FBD of cylinder:-

N_A is normal reaction of cylinder with ground.

Assume :- block of height 6 cm is of negligible mass.



ΣF_x = Fcos(45°) - $N_B$cos(56.44°) = 0 …..(1) (taking right as +ve)

ΣF_y = N_A – 6000 + Fsin(45°) + $N_B$sin(56.44°) =0 (taking upwards as +ve)

= N_A + Fsin(45°) + $N_B$sin(56.44°) = 6000 ……..(2)

Moment of all forces about point B is zero.

Moment is equal to force * perpendicular distance of the line of force vector to the point.

All distance are in 'cm' convert them in 'm' .

ΣM_B = 6000*(6√11/100) - N_A * (6√11/100) - Fsin(45°)(6√11/100) - Fcos(45°)*(30/100) + N_B*0 = 0

$N_A * (6\sqrt{11}/100) + F\sin(45°)(6\sqrt{11}/100) + F\cos(45°)*(30/100) = 1193.98$ ....(3)

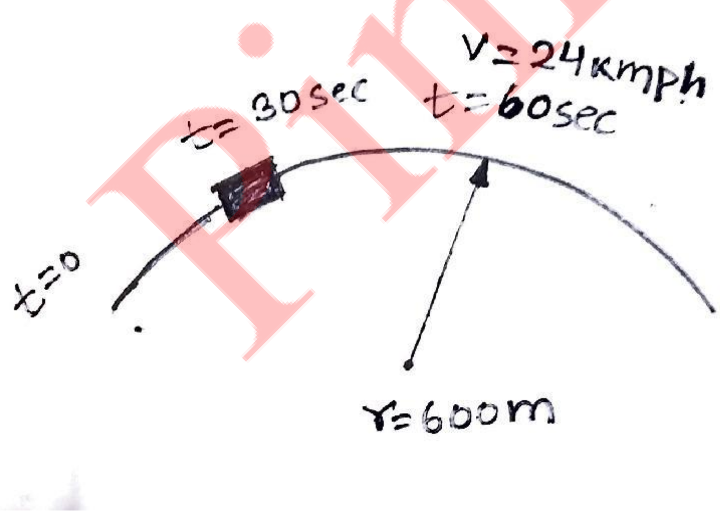
By eqauation 1, 2 and 3:-

F = 227.76 N (ANS)

c. **(6)**

At the instant t=0, a locomotive start to move with uniformly accelerated speed along a circular curve of radius r=600 m and acquires, at the end of the first 60 seconds of motion, a speed equal to 24kmph. Find the tangential and normal acceleration at the instant t=30 s.

Soln:-



At t = 30 sec :-

Normal acceleraltion $a_n = v^2/r$ (r = radius of curvature)

(v = 24kmph = 6.67m/s)

= 6.67/600

= 0.011m/s^2 (ANS)

Tangential acceleration a_t :-

$V = U + a_t*t$ (V=final velocity ,U=initial velocity,t=time taken)

$6.67 = 0 + a_t*30$

a_t = 0.22 m/s^2 (ANS)

4.)

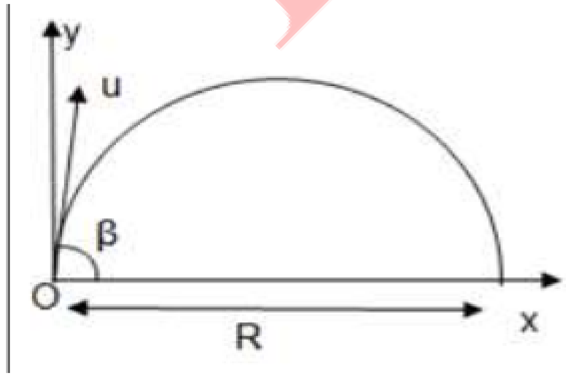
a. **(8)**

A particle is thrown with an initial velocity of 10 m/s at a 45° angle with horizontal. If another particle is thrown from the position at an angle 60° with the horizontal, find the velocity of the latter for the following situation:

(i) Both have the same range.

(ii) Both have the same time of flight.

Soln:-



Consider a particle performing projectile motion.

R – Horizontal Range

T – Total flight time

Considering vertical components of motion,

s = ut + at 2

0 = usin(β)*T- gT^2 , T = 2usin(β)/g

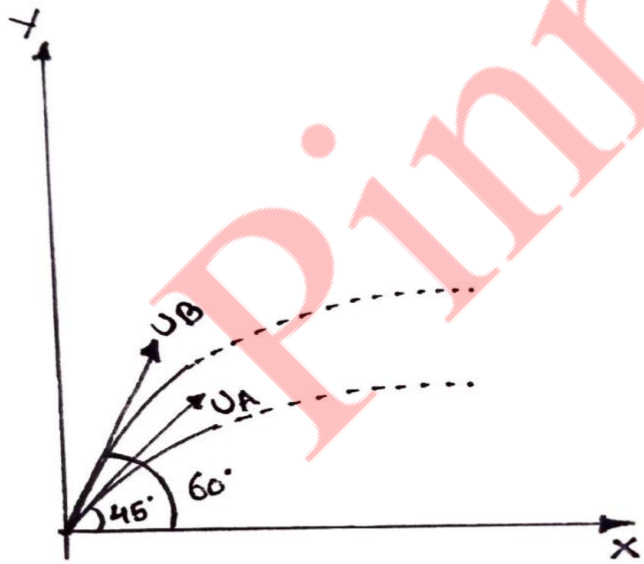
Considering horizontal components of motion,

s = ut + at 2

R = ucos(β)T + 0 ....(as acceleration in x direction is zero)

R = ucos(β) x 2usin(β)/g

R =(u^2sin(2β))/g



(I For same range :-

(UA)^2*sin(2*45°))/g = (UB)^2*sin(2*60°))/g

$10^2*\sin(90°)/g = (U_B)^2*\sin(120°)/g$

$U_B = 10.746$ m/s (ANS)

(II For same time of flight :-

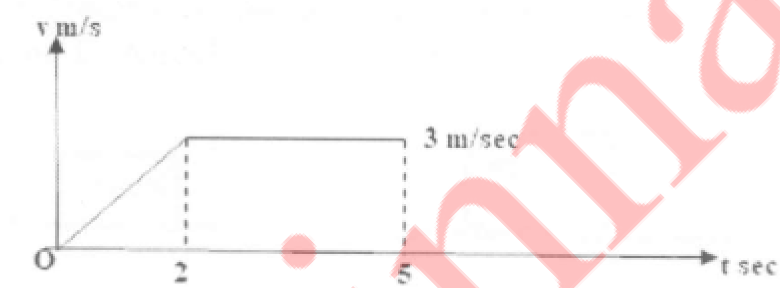
$2U_A\sin(45°)/g = 2U_B\sin(60°)/g$

$2*10*\sin(45°)/g = 2*U_B*\sin(60°)/g$

$U_B = 8.16$ m/s (ANS)

b. **(6)**

The motion of a particle is represented by the velocity-time diagram as shown in the graph shown below. Draw acceleration-time and displacement–time graphs.



Soln:-

(0-2)sec:-

Velocity is uniformly changing. So, acceleration will be contant and

a= (final velocity – initial velocity) / (final time – initial time)

= (3-0)/(2-0)

= 1.5m/s^2

For this time period curve of acceleration time graph will be 0° curve showing a constant value 1.5 m/s^2.

Displacement curve will be of 2° as velocity is uniform.

X2-X1 = area under velocity time graph for (0-2)sec

X2-0=½*3*2 , X2=3m

(2-5)sec:-

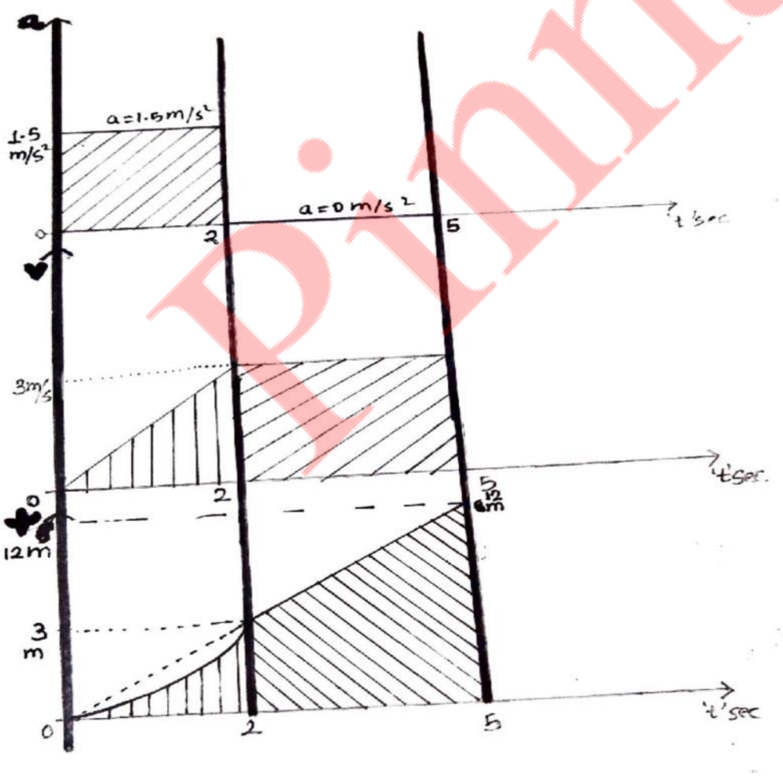
Velocity is constant for this time period.

So, acceleration time curve will be of 0° showing value 0m/s^2.

Displacement time graph curve will be of 1° as velocity is constant.

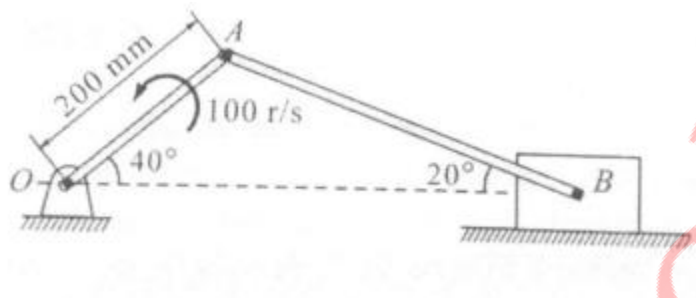
X5-X2 = area under velocity time graph for (2-5)sec

X5-3 = 3*3 , X5 = 12m



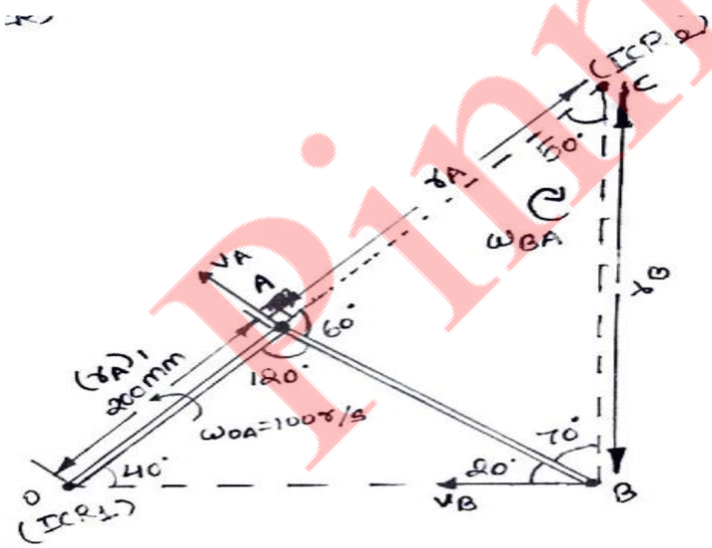
c. **(6)**

In the reciprocating engine mechanism shown in fig. The crank OA of the length 200mm rotates at 100 rad/sec. Determine the angular velocity of the connecting rod AB and the velocity of the piston at B.



Soln:-

Given:- 1. Motion of rod OA is considered for that ICR is point O and



velocity of point A is vA perpendicular to rod OA , ω_{OA} = 100rad/sec, radius of center of rotation of A is rA = 200mm or 0.2m(radius of center of rotation of point is measured from ICR).

2. Motion of rod AB from B to A , ICR of this motion c ,velocity of point A and B are vA nad vB, for this motion radius of center of rotation for A and B are rA and rB resp. And angular velocity is ω_{BA} = ?.

To find:- **ω_{BA} = ? , vB=?**

Calculation:-

1. Motion of rod OA:-

vA = ω_{OA}*rA

= 100*0.2

= 20m/s

2. Motion of rod AB:-

ω_{AB} = vA/rA = vB/rB

In ΔAOB,

By sine rule,

OA/sinB = AB/sinO = BO/sinA

(rA)1/sin20°=AB/sin40°

0.2/sin20°=AB/sin40°

AB=0.375m

In ΔABC,

By sine rule,

AB/sinC = BC/sinA = CA/sinB

0.375/sin50°= rB/sin60°=rA/sin70°

rA=0.460m

rB=0.424m

ωAB = vA/rA = vB/rB

ωAB = vA/rA

= 20/0.460

= 43.47r/s (ANS)

vB = **ωAB*rB**

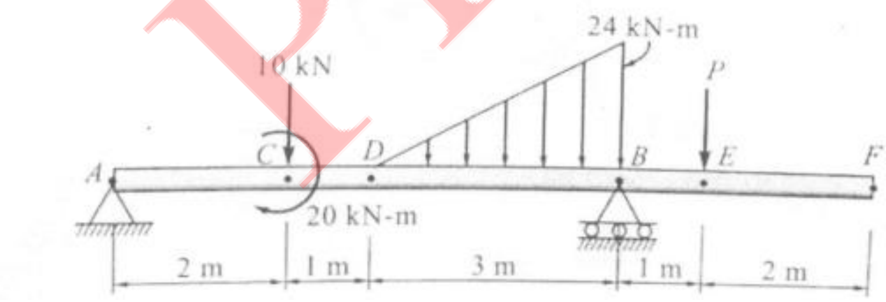
= 43.47*0.424

= 18.43 m/s (ANS)

5.)

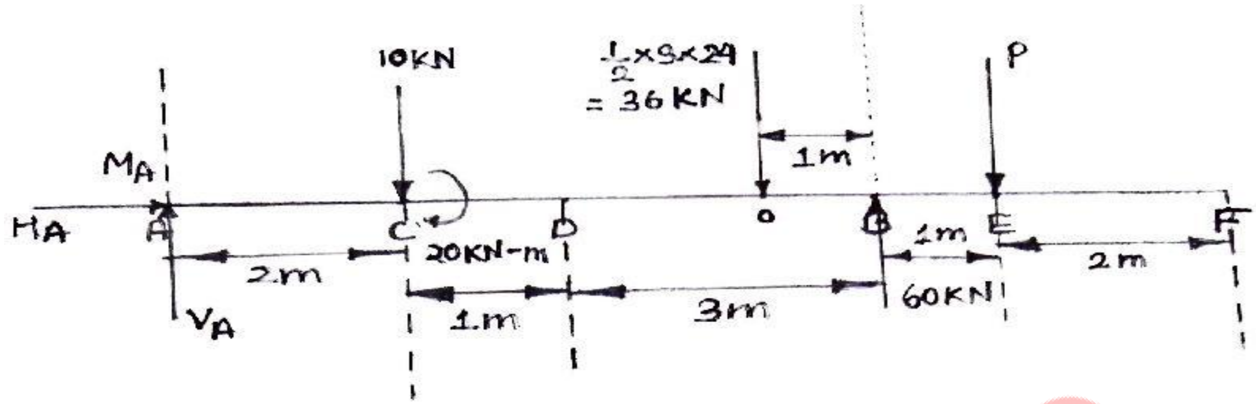
a. (8)

Find the support reaction at A and forces P if reaction at B is 60 kN for the beam loaded as shown in Figure below.



Soln:-

Given:- The figure 1st and 2nd .



$\sum Fx = 0$

$H_A = 0$

$\sum Fy = -10 + 60 - 36 + V_A - P = 0$

$V_A - P = 14$ ....(1)

$\sum M_A = -(Px7) + (60x6) - (10x2) - (36*5) = 0$ (moment of forces along A is zero)

(taking anti-clockwise as +ve)

P = -22.85 kN or 22.85 kN (upwards) ....(2) (given direction of 'P' was wrong)

From (1) and (2)

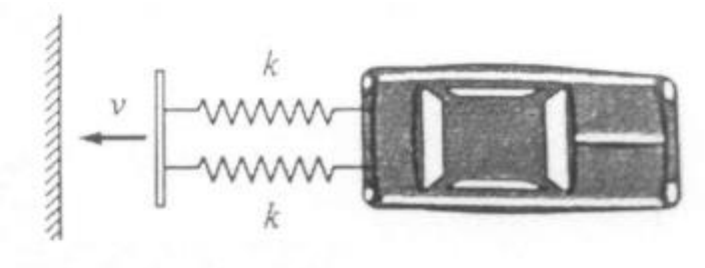
$V_A = 36.85$ kN

The magnitudes of force V_A , $H_A = 0$ and reaction P are 36.85kN and 22.85 kN respectively. (ANS)

b. **(6)**

A 1200 kg car has a light bumper supported horizontally by two springs of stiffness 15kN/m. Determine the initial speed of impact

with the fixed wall that causes 0.2 m compression. Neglect Friction.



Soln:-

Given :- Car of mass 'm' = 1200kg , spring of stiffness 'k' = 15*10^3N/m, initial and final compression are x1 and x2,initial and final speed are v1 and v2.

To find:- v1=?

Calculation:-

T1 = Initial kinetic energy = ½*m*(v1)^2 = ½*1200*(v1)^2 = 600*(v1)^2

T2 = Final kinetic energy = ½*m*(v2)^2 = ½*1200*0 (final velocity v2=0 as it bumps)

= 0

Work done by spring = ½*k_{eq}*((x1)^2-(x2)^2) (resultant stiffness k_{eq})

Two springs of same stiffness are parallel.

So, resultant stiffness = 2k

x1 = 0m (no deflection) , x2= 0.2m

= ½*2k*(0 – (0.2)^2) = ½*2*(15*10^3)*(-(0.2)^2) = -600J

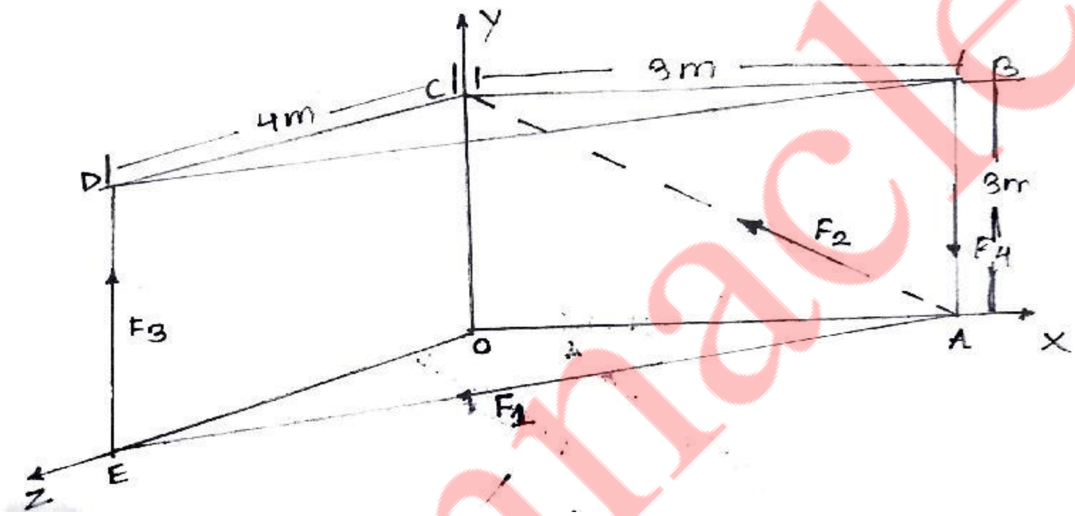
By work energy theorem:-

T1 + work done = T2

600*(v1)^2 + (-600) = 0

v1 = 600/600 = 1m/s (ANS)

c. (6) Determine the resultant force of the force system shown in figure where F1=150N, F2=120N, F3=200N and F4=220N.



forces are given as:

$|\bar{F}1|$=150N, $|\bar{F}2|$=120N ,$|\bar{F}3|$=200N, $|\bar{F}4|$=220N

WE CAN GET VELOCITY VECTOR BY MULTIPLYING THEM WITH UNIT VECTOR IN THEIR DIRECTION FROM THE DIAGRAM.

$\bar{F}1 = 150(3i + 4k)/\sqrt{3^2 + 4^2}$

$= 90i + 120k$

$\bar{F}2 = 120(3i + 3j)/\sqrt{3^2 + 3^2}$

$= 60\sqrt{2}i + 60\sqrt{2}j$

$\bar{F}3 = 200(3j + 4k)/\sqrt{3^2 + 4^2}$

$= 120j + 160k$

$\bar{F}4 = 220(3i - 3j)/\sqrt{3^2 + 3^2}$

$= 110\sqrt{2}i - 110\sqrt{2}j$

$\bar{R} = \Sigma\bar{F}$

$= (R_X)i + (R_Y)j + (R_Z)k$

$= \Sigma F_X + \Sigma F_Y + \Sigma F_Z$

$= (90+170\sqrt{2})i + (120-50\sqrt{2})j + (280)k$

$|\bar{R}| = \sqrt{(R_X)^2 + (R_Y)^2 + (R_Z)^2}$

$= \sqrt{(90+170\sqrt{2})^2 + (120-50\sqrt{2})^2 + (280)^2}$

$= 435.894$ N

$\cos\theta_x = R_X/R = (90+170\sqrt{2})/435.894 = 0.758$ (direction)

$\cos\theta_y = R_Y/R = (120-50\sqrt{2})/435.894 = 0.113$ (direction)

$\cos\theta_z = R_Z/R = (280)/435.894 = 0.642$ (direction)

$\Theta_x = 40.71°$

$\Theta_y = 83.51°$

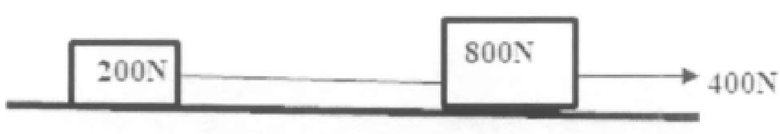
$\Theta_z = 50.05°$

6.)

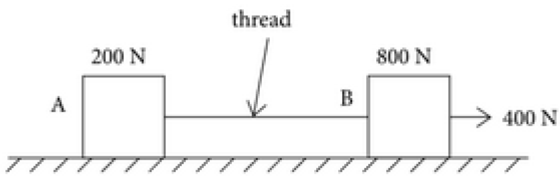
a. **(8)**

Two bodies A and B are connected by a thread and move along a rough horizontal plane (μ=0.3) under the action of 400N force

applied to the body as shown in Fig. Determine the acceleration of the two bodies and the tension in the thread using D' Alembert’s principle.

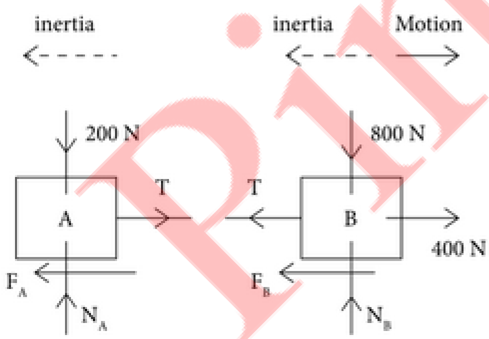


Soln:-



Given:- u=0 and μ = 0.3 .

The free body diagram of A and B is as shown below. Let blocks A and B are accelerated by acceleration a_A and a_B respectively.



As both the bodies move to right side, their inertia will act opposite to motion as shown. Using D’Alembert’s principle for body B,

Net force causing motion + inertia of body = 0

Net force causing motion = 400 – F_B - T

Inertia force of body = $(-800*a_B/g)$

$400 – F_B - T -800*a_B/g = 0$ .....(1)

And $N_B = 800$ N

Kinetics of particle:-

Thus equation (1) may be written as

$400 – \mu N_B - T = 800*a_B/g$

$400 – 0.3*800 - T = 800*a_B/g$

$160 - T = 800*a_B/g$ ......(2)

Similarly , Using D'Alembert's principle for body B,

$T – F_A – 200*a_A/g = 0$

$T – 0.3N_A = 200*a_A/g$

and, $N_A = 200$ N

$T – 60 = 200*a_A/g$ ....(3)

By putting $a_A = a_B = a$ and solving equation (2) and (3) :-

$160 - T +T– 60 = 200*a/g + 800*a/g$

$100 = 1000*a/g$ (g=9.81m/s^2)

a = 0.981 m/s^2

Substituting the value of 'a' in equation (3)

$T – 60 = 200*0.981/g$ (g=9.81m/s^2)

T = 80N (ANS)

$a_A = a_B = a = 0.981 m/s^2$ (ANS)

b. **(6)**

Train A starts with uniform acceleration of 0.5m/s^2 and attains a speed of 90km/hr which subsequently remains constant. One minute after it starts, another train B starts on a parallel track with a uniform acceleration of 0.9m/s^2 and attains a speed of 120km/hr. How much time does train B take to overtake train A.

Soln:-

TRAIN A:-

Initial velocity at A 'uA' = 90km/hr = 25m/s

Accleration of A 'aA' = 0.5m/s^2

Time taken 'tA'= t sec

Distance covered = 'sA' m

TRAIN B:-

Initial velocity at B 'uB' = 120km/hr = 33.33m/s

Accleration of B 'aB' = 0.5m/s^2

Time taken 'tB'= t-60 sec (Because train 'B' is starting 1 min late then Train 'A')

Distance covered = 'sB' m

Time of overtake will come when distance covered by both the trains are same.

Hence,

$sA = sB$

$(uA)*(tA) + ½*(aA)*(tA)^2 = (uB)*(tB) + ½*(aB)*(tB)^2$

$25*t + ½*0.5*t = 33.33*(t-60) + ½*0.9*(t-60)^2$

$25*t + 0.25*t^2 = 33.33*t – 1999.8 + 0.45*(t^2 – 120*t + 3600)$

$0.20*t^2 – 45.67*t – 379.8 = 0$

After solving the quadratic equation:-

t= 236.38 sec or –8.03 sec

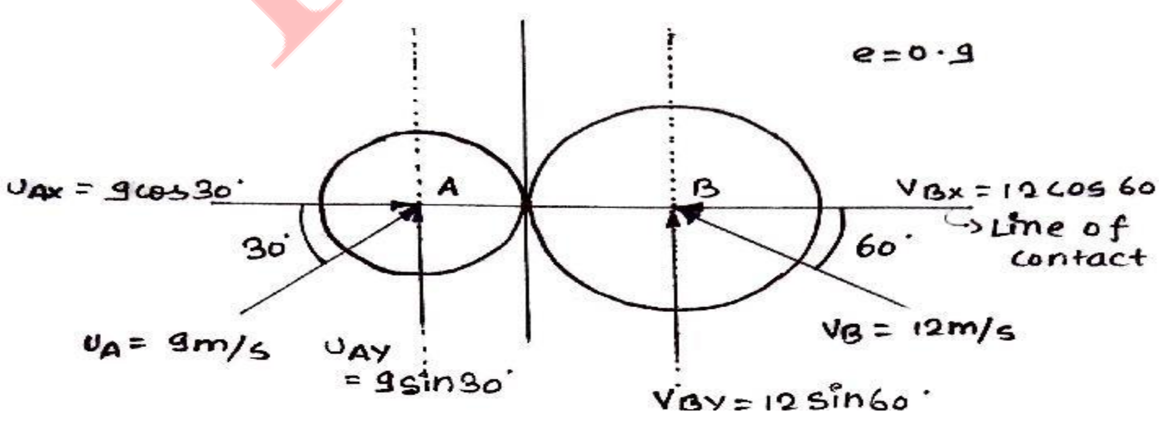
But time can't be negative so

T = 236.38 sec or 4min 56sec (ANS)

c. **(6)**

The magnitude and direction of the velocities of two identical spheres having frictionless surfaces are shown in figure below. Assuming coefficient of restitution as 0.90, determine the magnitude and direction of the velocity of each sphere after the impact. Also find the loss in kinetic energy

SOLUTION :-



Let mass of both identical bodies = m kg

Coefficient of restitution 'e' = 0.90

This impact is oblique collision.

U_{AX} = 7.794m/s (rightwards) U_{BX} = 6m/s (leftwards)

U_{AY} = 4.5m/s U_{BY} = 10.39m/s

$U_{AY} = V_{AY}$= 4.5m/s (UPWARDS)

$U_{BY} = V_{BY}$= 10.39m/s (UPWARDS)

By LCM :-

IM = FM

$m(7.794) + m(6) = m(V_{AX}) + m(V_{BX})$

$(V_{AX}) + (V_{BX}) = 13.794$ ....(1)

$e = (V_{BX} - V_{AX}) / (U_{AX} - U_{BX})$

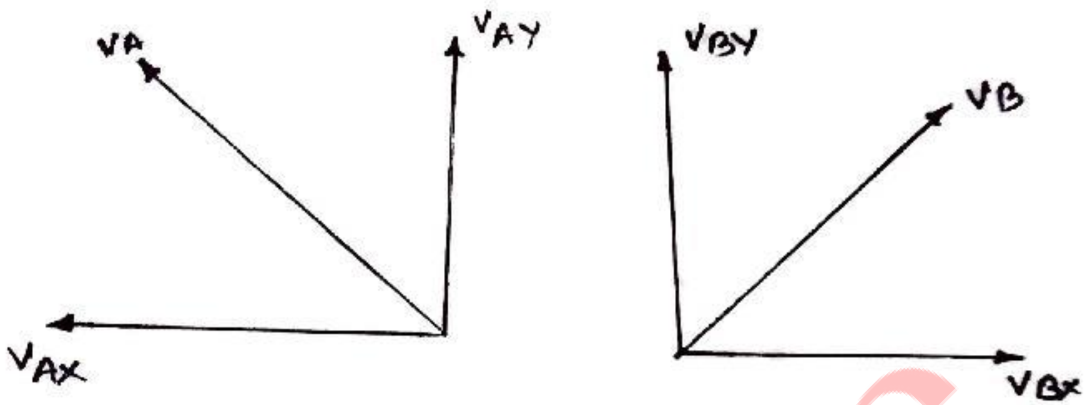
$V_{BX} - V_{AX} = 0.90*(7.794-(-6))$

$= 12.41$ ....(2)

By solving equation (1) and (2)

V_{BX} = 13.102m/s (rightwards)

V_{AX} = 0.692m/s (leftwards)



$V_A = \sqrt{(V_{AX})^2 + (V_{AY})^2} = \sqrt{(4.5)^2 + (0.692)^2}$

= 4.55 m/s (ANS)

$\Theta_A = \tan^{-1}(V_{AY} / V_{AX})$

= 81.25° (ANS)

$V_B = \sqrt{(V_{BX})^2 + (V_{BY})^2} = \sqrt{(10.39)^2 + (13.102)^2}$

= 16.72 m/s (ANS)

$\Theta_B = \tan^{-1}(V_{BY} / V_{BX})$

= 38.414° (ANS)

Loss in kinetic energy

= ½*(m1*m2/(m1+m2))*(1-e^2)*(u1cosα1 - u2cosα2)^2

= ½*(m*m/(2m))*(1-(0.9)^2)*(9cos(30°)-12cos(60°))^2

= **0.085 J (ANS)**